Vehicle Parameter Design Based on Nonlinear Bifurcation Analysis of Hunting Motion of a Wheelset

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ABSTRACT: This paper deals with a nonlinear hunting motion of a railway wheelset. A hunting motion has characteristics of Hopf bifurcation. Hopf bifurcation is classified with sub critical Hopf bifurcation and super critical Hopf bifurcation. In case a sub critical Hopf bifurcation, an amplitude of a limit cycle of a hunting motion exists under a critical speed of a railway vehicle. Therefore, if a railway vehicle has bogie parameters of a sub critical Hopf bifurcation characteristics, a hunting motion of the vehicle might occur by some disturbance under a critical speed. In this paper, the characteristics of a sub critical Hopf bifurcation of hunting motion is considered. Using center manifold theory, equations of motion are reduced to differential equations of normal form. Moreover, an amplitude of a limit cycle is considered from the differential equations of normal form.

1 INTRODUCTION

In dynamics of railway vehicles, hunting motion has been to researched by many researchers for a long time[1], [2]. By using these results, parameters of a vehicle bogie are designed based on the linearized equations of motion. However, a critical speed obtained from the linearized equations of motion of a railway vehicle is a difference from an actual critical speed. On the other hand, railway vehicle dynamics with nonlinearity has been researched. In these researches, there is a topic about a bifurcation phenomenon of a hunting motion of a railway vehicle[3]-[10]. In a hunting motion of a railway vehicle, a Hopf bifurcation occurs in a vehicle. There are two types in the Hopf bifurcation. One is a sub critical Hopf bifurcation. Another is a super critical Hopf bifurcation. When design parameters for a bogie is changed, a hunting motion changed between a sub critical Hopf bifurcation and a super critical Hopf bifurcation. In a case of a sub critical Hopf bifurcation, there is a possibility to start a hunting motion by a disturbance under a critical speed. From this point of view, it is desirable to use design parameters that cause a super-critical Hopf bifurcation rather than design parameters that cause a sub-critical Hopf bifurcation.

In this paper, characteristics of Hopf bifurcation are considered by adjusting a design parameter of a bogie. First, equations of motion including nonlinear terms for a bogie with a single wheelset were used. These equations of motion were reduced using the centre manifold theory. Next, differential equations of a normal form for the bifurcation phenomena were derived using a nonlinear coordinate transformation. The basic consideration for the design parameters for a bogie with a single wheelset were carried out from the amplitude and the bifurcation types of the limit cycle.

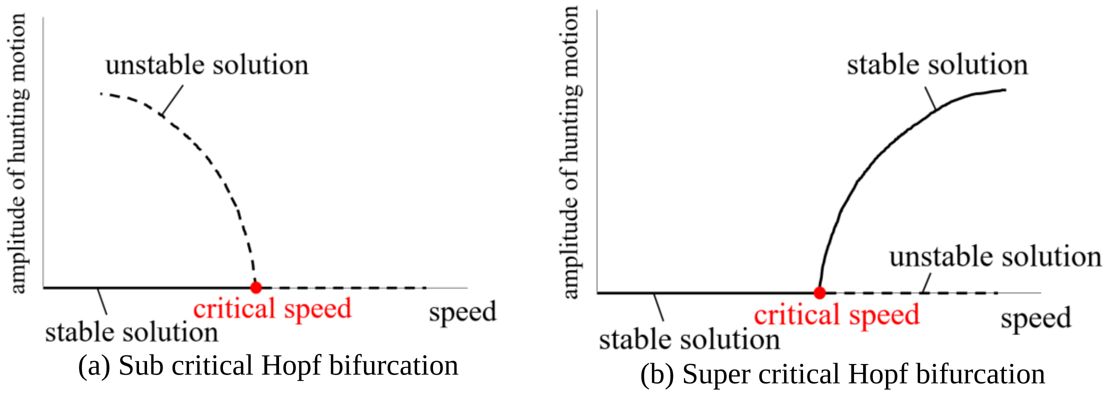

Figure 1 Hopf bifurcation

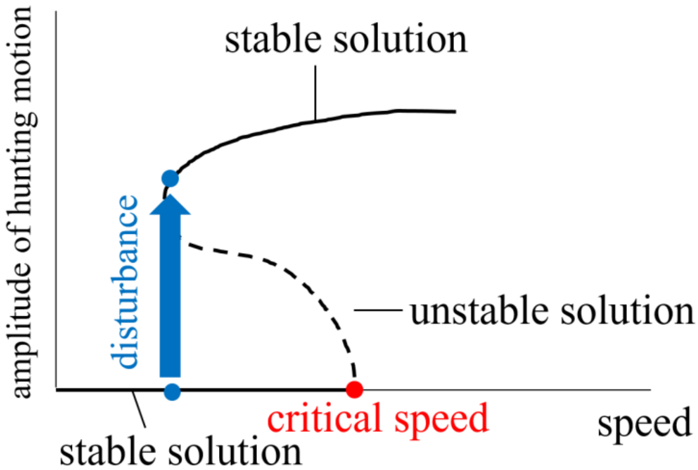

Figure 2 Occurrence of a hunting motion due to disturbances under a critical speed

2 VEHICLE MODEL

A vehicle model is shown in Figure 1. In this paper, a single axle truck model is used for a following analysis. In this figure, v denotes a vehicle speed, r_0 denotes a radius of a wheel, a denotes a half of a track, L denotes a half of a distance of axle boxes, γ denotes a conicity, k_x denotes a spring stiffness in a longitudinal direction and k_y denotes a spring stiffness in a lateral direction.

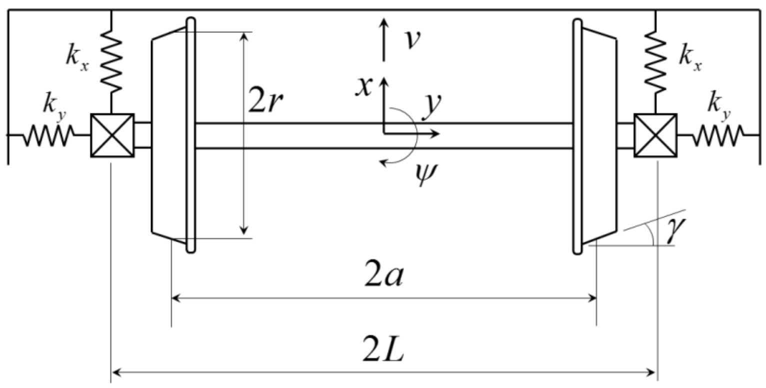

Figure 3 Vehicle model

Equations of motion of this model are described as follows:

$$\left.\begin{aligned} m\frac{d^2y}{d\hat{t}^2}+\frac{2f_{22}}{v}\frac{dy}{dt}+2k_y y-2f_{22}\psi+\alpha_{yyy}y^3+\alpha_{yy\psi}y^2\psi+\alpha_{y\psi\psi}y\psi^2+\alpha_{\psi\psi\psi}\psi^3=0 \\ I\frac{d^2\psi}{dt^2}+\frac{2a^2f_{11}}{v}\frac{d\psi}{dt}+\frac{2f_{11}a\gamma}{r}y+2k_xL^2\psi+\beta_{yyy}y^3+\beta_{yy\psi}y^2\psi+\beta_{y\psi\psi}y\psi^2+\beta_{\psi\psi\psi}\psi^3=0 \end{aligned}\right\} \quad (1)$$

Here, m denotes a mass of the wheelset, I denotes a moment of inertia of the wheelset, α_{yyy}, $\alpha_{yy\psi}$, $\alpha_{y\psi\psi}$, $\alpha_{\psi\psi\psi}$, β_{yyy}, $\beta_{yy\psi}$, $\beta_{y\psi\psi}$ and $\beta_{\psi\psi\psi}$ are coefficients for nonlinear terms.

3 NONLINEAR ANALYSIS OF A HUNTING MOTION

These equations of motion were reduced to the normal form using the center manifold theory. The reduced equations of motion using polar coordinates are described as follows:

$$\left.\begin{aligned} \dot{r}&=\kappa_{1101r}\varepsilon r+\kappa_{1210r}r^3 \\ \dot{\theta}&=\lambda_{1i}+\kappa_{1101i}\varepsilon+\kappa_{1210i}r^2 \end{aligned}\right\} \quad (1)$$

Here, r denotes an amplitude, θ denotes a phase, ε denotes a speed variance from a critical speed. From these equations, an amplitude of the hunting motion is derived as following:

$$r_f=\sqrt{-\frac{\kappa_{1101r}}{\kappa_{1210r}}\varepsilon} \quad (2)$$

4 RESULTS

In the case a design parameter was changed, amplitude of the hunting motion was considered. When the amplitude of the hunting motion r_f is the real number, the amplitude of the hunting motion exists. Therefore, if $\kappa_{1101r}/\kappa_{1210r}>0$ is given, r_f become the real number in $\varepsilon<0$ condition. This is a sub critical Hopf bifurcation. On the other hand, if $\kappa_{1101r}/\kappa_{1210r}<0$ is given, r_f become the real number in $\varepsilon>0$ condition. This is a super critical Hopf bifurcation. When a primary spring stiffness is changed from 1×10^6 to 1×10^9 N/m, the amplitude of the hunting motion is shown in Figure2. From this result, the amplitude of the hunting motion occurs only under $\varepsilon<0$ condition. Therefore, a sub critical Hopf bifurcation occurs in any primary spring stiffness conditions.

5 CONCLUSIONS

In this paper, in order to change the characteristics of a Hopf bifurcation from the sub-critical Hopf bifurcation to the super-critical Hopf bifurcation, characteristics of Hopf bifurcation were considered by adjusting a design parameter of a bogie. First, equations of motion including nonlinear terms for a bogie with a single wheelset were used. These equations of motion were reduced using the centre manifold theory. Next, differential equations of a normal form for the bifurcation phenomena were derived using a nonlinear coordinate transformation. The basic consideration for the design parameters for a bogie with a single wheelset were carried out from the amplitude and the bifurcation types of the limit cycle.

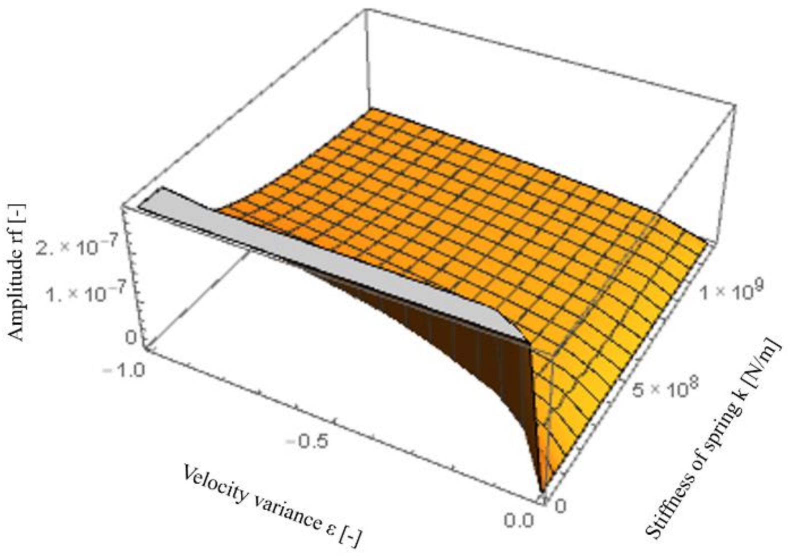

Figure 4 Relationship in velocity, spring stiffness and hunting amplitude

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